Education

KwaZulu-Natal Department of Education

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2018

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 11 pages, 1 information sheet and an answer book of 23 pages.

INSTRUCTIONS AND INFORMATION

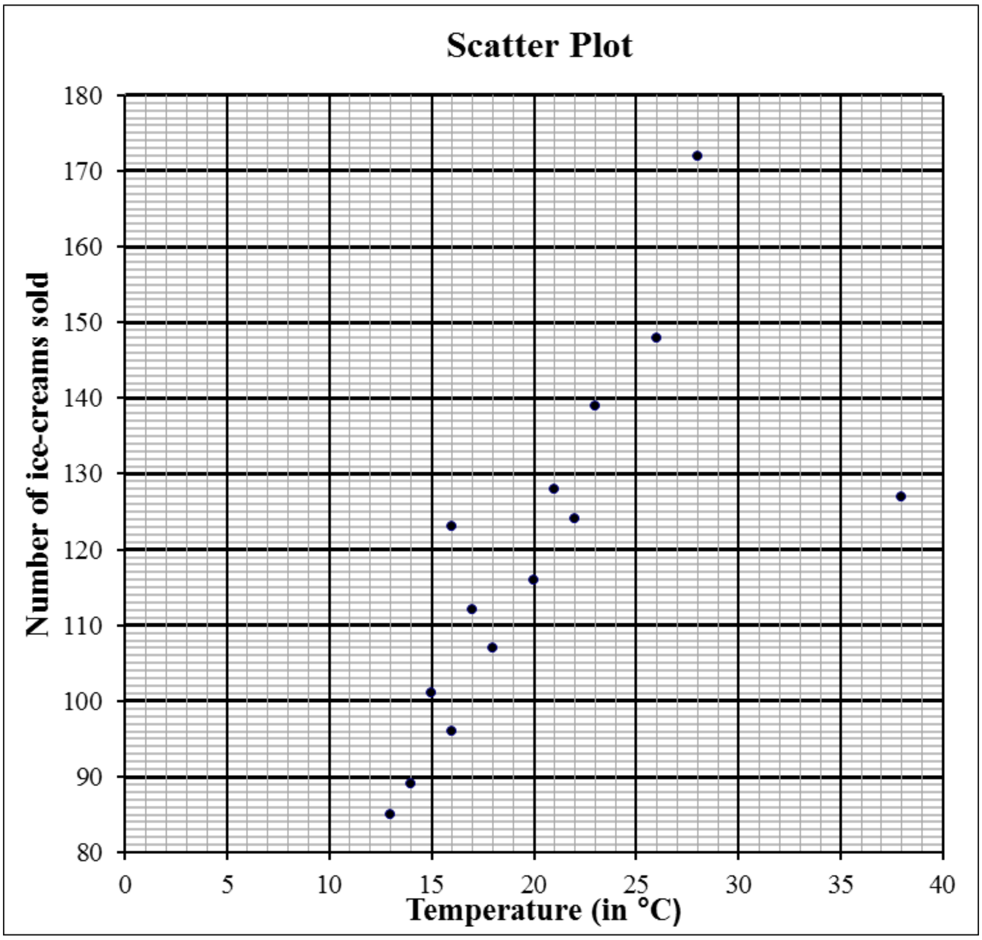
Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 1

Mrs Simakuhle sells ice cream to high school learners in her neighbourhood. The sales were analysed over 14 randomly selected days. Each sale was compared with the recorded maximum on the day. This information is reflected in the table below.

Temperature (in °C)	15	21	17	22	20	16	16	23	38	13	28	14	26	18
Number of ice creams sold per day	101	128	112	124	116	96	123	139	127	85	172	89	148	107



1.1 Comment on the trend of the data. (1)

1.2 Identify the outlier in the data set. (1)

1.3 Determine the equation of the least squares regression line excluding the outlier. (3)

1.4 Predict the number of ice creams sold per day if the maximum air temperature is 24°C (2)

[7]

QUESTION 2

The following weights (in kgs) were recorded from 15 randomly selected weight lifters at a certain gymnasium.

79	80	85	88	89	89	92	94	101	105	106	107	108	112	113

2.1 Calculate the mean weight of the weight lifters. (2)

2.2 Calculate the standard deviation of the recorded weights. (2)

2.3 How many weight lifters would be classified in the feather weight division if you have to weigh less than one standard deviation from the mean weight? (2)

2.4 Draw a box and whisker diagram for the above data. (5)

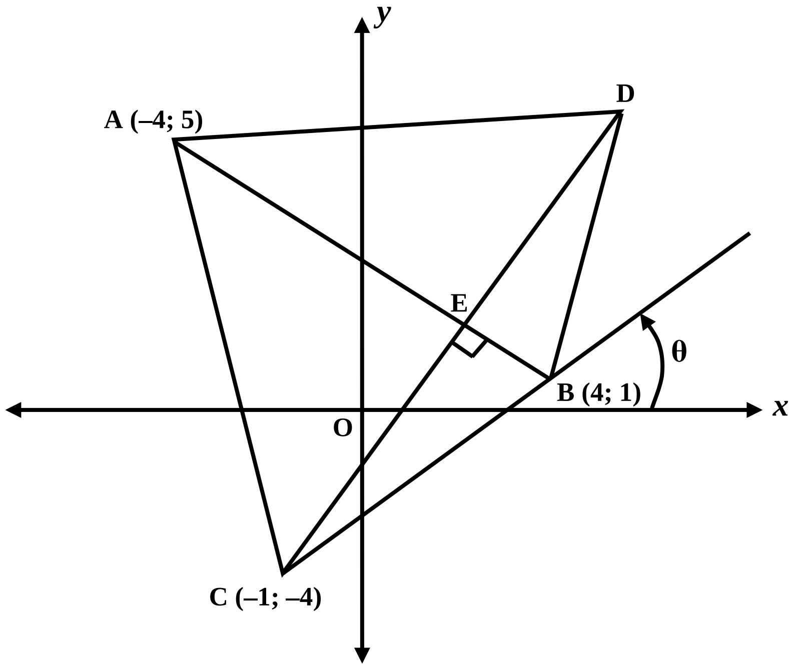
2.5 Calculate the IQR. (2)

2.6 Comment on the spread of the data. (1)

[14]

QUESTION 3

In the diagram below, A (– 4; 5); C (–1; – 4) , B (4; 1) and D are the vertices of a quadrilateral.
E is the midpoint of CD and the point of intersection of the diagonals of ABCD.
AB $\perp$ CED. θ is the angle of inclination of line CB.



3.1 Determine

3.1.1 the gradient of AB. (2)

3.1.2 the equation of AB. (2)

3.1.3 the equation of CD. (3)

3.1.4 the coordinates of E. (5)

3.1.5 the equation of the line parallel to BC and passing through A. (3)

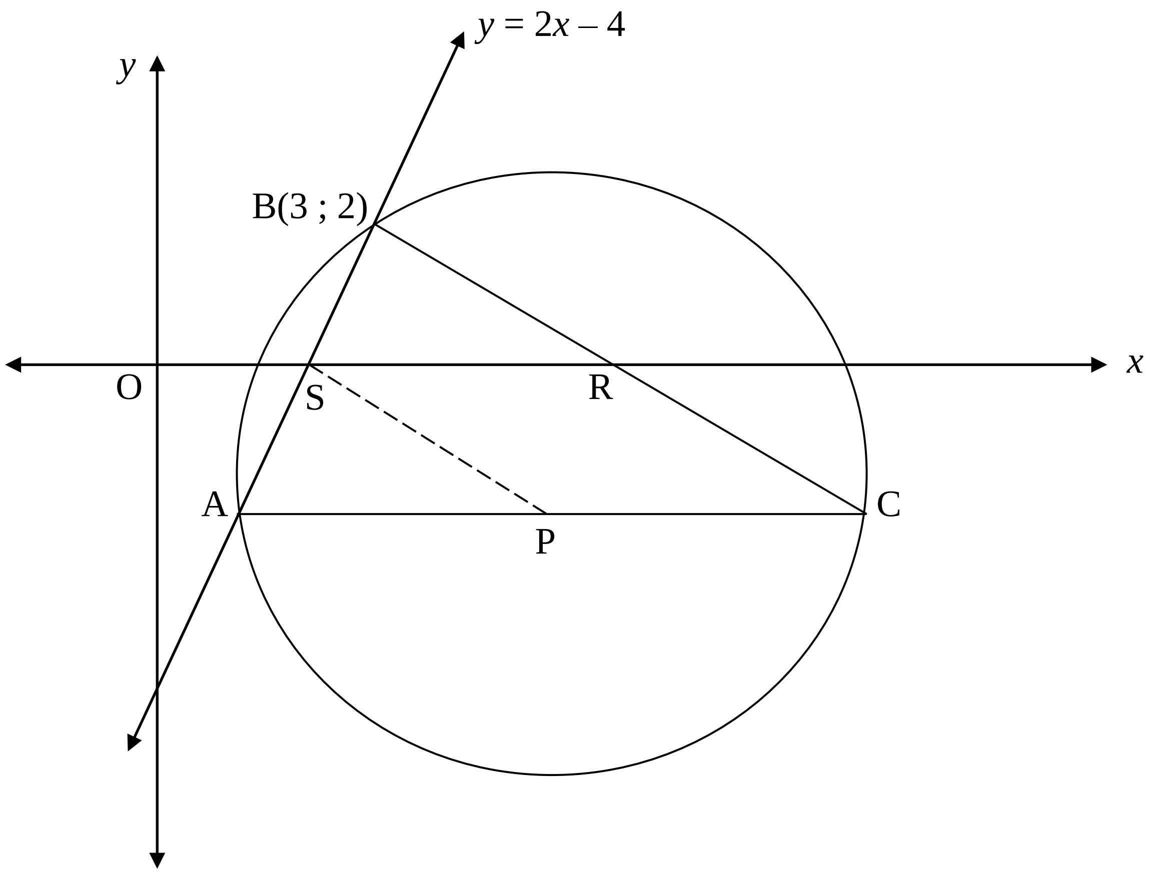
3.2 Calculate the value of θ. (2)

3.3 Calculate the area of Δ AEC. (4)

[21]

QUESTION 4

In the figure , the straight line $y = 2x - 4$ and the circle $(x - 6)^2 + (y + 2)^2 = 25$ intersect at A and B(3; 2). P is the centre of the circle and APC is the diameter. Also R is the x – intercept of line BC and S is the x – intercept of AB.



4.1 Write down the coordinates of the centre of the circle, P. (2)

4.2 Calculate the coordinates of S. (2)

4.3 Determine the equation of the line BC. (4)

4.4 Determine the equation of the circle with centre R and passing through B and C. (5)

4.5 Show that AC // SR. (5)

[18]

QUESTION 5

5.1 Given:

$4\tan\alpha + 5 = 0,\ \alpha \in (0^\circ; 180^\circ)$. Evaluate without using a calculator:

$\sqrt{41}\ \cos\alpha - 4\sin(-150^\circ) . \cos 180^\circ$ (5)

5.2 Simplify, without the use of a calculator.

5.2.1 $\dfrac{\cos 99^\circ}{\cos 33^\circ} - \dfrac{\sin 99^\circ}{\sin 33^\circ}$ (6)

5.2.2 $\dfrac{\cos 140^\circ - \sin(90^\circ - \theta)}{\sin 130^\circ + \cos(-\theta)}$ (5)

5.3 Prove the identity:

$\dfrac{2\sin^2 x}{2\tan x - \sin 2x} = \dfrac{\cos x}{\sin x}$ (6)

5.4 Determine the general solution of the following equation:

$8\sin\theta\cos\theta = -2\sqrt{3}$ (7)

[29]

QUESTION 6

Given: $f(x) = 3\cos x$ and $g(x) = \tan 2x$ for $x \in [-45^\circ ; 225^\circ]$

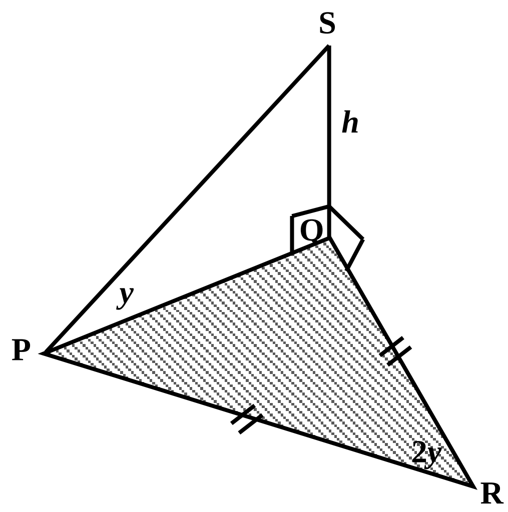
6.1 Sketch on the same set of axes the graphs of f and g. Clearly indicate any asymptotes using dotted lines. (8)

6.2 One solution of the equation $3\cos x = \tan 2x$ is 34°. Use your graph, to determine any other solutions in the given interval. (2)

[10]

QUESTION 7

In the diagram QS is a vertical pole. P and R are points in the same horizontal plane as Q such that QP = QR. The angle of elevation of the top of the pole S from P is y.
Also SQ = h and $P\hat{R}Q = 2y$.

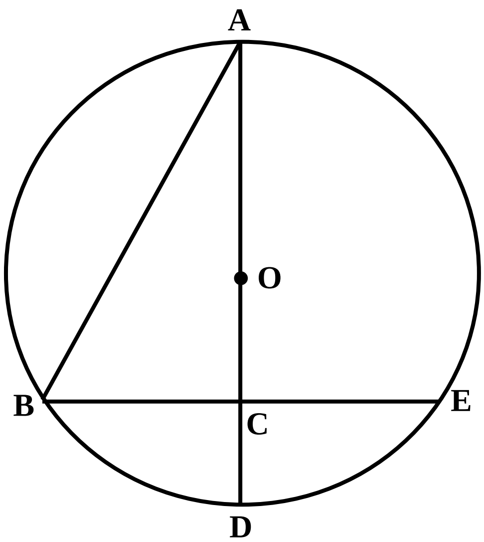


Prove that:

$$PR = \frac{h \,.\, \cos^2 y}{\sin y \,.\, \sin 2y}$$ **[6]**

QUESTION 8

In the diagram below, AOCD is a diameter of the circle with centre O and chord BE = 30 cm.
AOCD $\perp$ BE and OC = 2CD.



Calculate with reasons:

8.1 BC (2)

8.2 If CD = *a* units, determine OC in terms of *a*. (1)

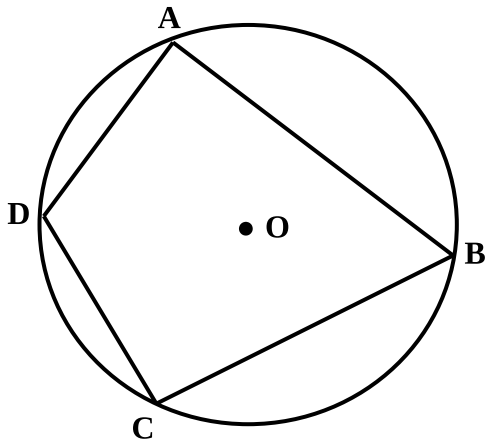
8.3 Calculate OB. (1)

8.4 AB (correct to one decimal place). (3)

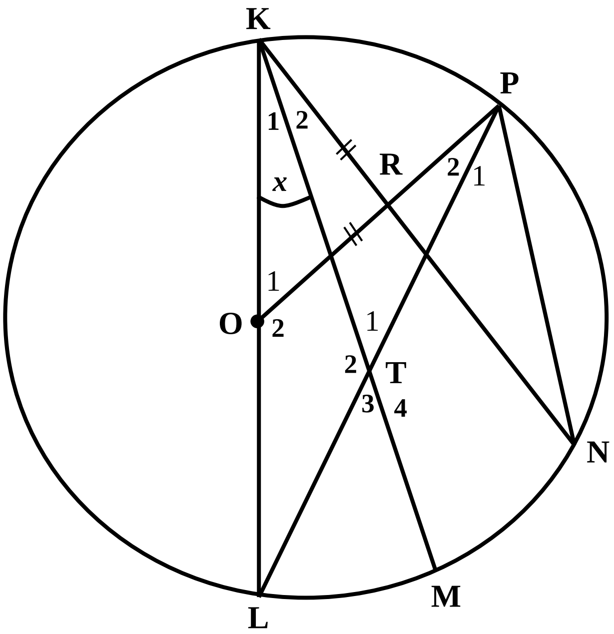
8.5 the radius of the circle CAB. (2)
[9]

QUESTION 9

9.1 In the diagram below, ABCD is a cyclic quadrilateral of the circle with centre O.
Use the diagram to prove the theorem which states that $\hat{B}+\hat{D}=180°$. (5)



9.2 KOL is the diameter of the circle KPNML having centre O. R is the point on chord KN, such that KR = RO. OR is produced to P. Chord KM bisects $L\hat{K}N$ and cuts LP in T.
$K_1 = x$.



Prove with reasons that:

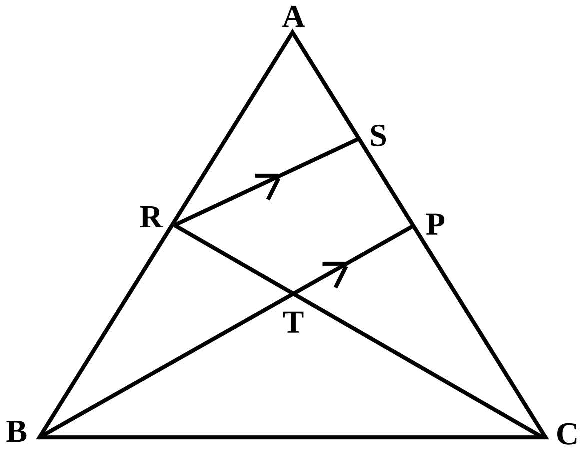
9.2.1 TK = TL (5)

9.2.2 KOTP is a cyclic quadrilateral. (3)

9.2.3 PN // MK (3)
[16]

QUESTION 10

In ΔABC, R is a point on AB. S and P are points on AC such that RS // BP. P is the midpoint of AC. RC and BP intersects at T. $\frac{AR}{AB} = \frac{3}{5}$.



Calculate with reasons, the following ratios:

10.1 $\frac{AS}{SC}$ (3)

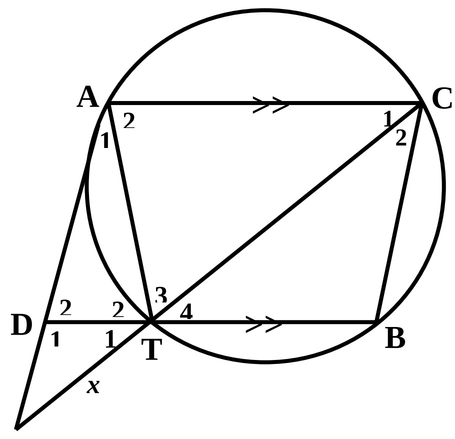
10.2 $\frac{RT}{TC}$ (2)

10.3 $\frac{\Delta ARS}{\Delta ABC}$ (3)

[8]

QUESTION 11

In the diagram alongside,
ACBT is a cyclic quadrilateral.
BT is produced to meet tangent AP on D.
CT is produced to P. AC // DB.



11.1 Prove that $PA^2 = PT \cdot PC$ (5)

11.2 If PA = 6 units, TC = 5 units and PT = x, show that $x^2 + 5x - 36 = 0$. (2)

11.3 Calculate the length of PT. (2)

11.4 Calculate the length of PD. (3)

[12]

TOTAL MARKS: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $\hat{y} = a + bx$

$$b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$